Limited belief propagation and contingent thinking

Andrew Ellis and Ran Spiegler
LSE and UCL+TAU

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Example: interdependent value auction

- Textbook (Osborne 2004) 2 player, second-price, common value auction of single good
- types t_1, t_2 distributed independently
- Player i bids b_i $(b_i = b_i(t_i))$
- Player i gets good with probability x_i $(x_i = x_i(b_1, b_2))$
- Player i values good at $\frac{1}{2}t_1 + \frac{1}{2}t_2$ and pays $b_j x_i$

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From Osborne (2004):

To verify this claim, suppose that each type of player 2 bids in this way and type t_1 of player 1 bids b_1. To determine the expected payoff of type t_1 of player 1, we need to find the probability with which she wins, and both the expected price she pays and the expected value of player 2's signal if she wins.

Probability that player 1 wins: Given that player 2's bidding function is $(\alpha+\gamma)t_2$, player 1's bid of b_1 wins only if $b_1 \geq (\alpha+\gamma)t_2$, or if $t_2 \leq b_1/(\alpha+\gamma)$. Now, t_2 is distributed uniformly from 0 to 1, so the probability that it is at most $b_1/(\alpha+\gamma)$ is $b_1/(\alpha+\gamma)$. Thus a bid of b_1 by player 1 wins with probability $b_1/(\alpha+\gamma)$.

Expected price player 1 pays if she wins: The price she pays is equal to player 2's bid, which, *conditional on its being less than* b_1, is distributed uniformly from 0 to b_1. Thus the expected value of player 2's bid, *given that it is less than* b_1, is $\frac{1}{2}b_1$.

Expected value of player 2's signal if player 1 wins: Player 2's bid, given her signal t_2, is $(\alpha+\gamma)t_2$, so that the expected value of signals that yield a bid of less than b_1 is $\frac{1}{2}b_1/(\alpha+\gamma)$ (because of the uniformity of the distribution of t_2).

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Belief propagation in auction

- corresponds to **belief propagation** problem in CS
- computationally tractable algorithm uses the DAG

$$t_1 \to b_1 \to x_1 \leftarrow b_2 \leftarrow t_2$$

start with t_1 and b_1 and successively updates new variables

- procedure:
  1. compute $p(x_1|b_1)$ (no t_1; $x_1 \perp t_1|b_1$)
  2. compute $p(b_2|x_1, b_1)$ $(b_2 \perp t_1|(b_1, x_1))$
  3. compute $p(t_2|b_2)$ $(x_2, b_1, t_1 \perp t_2|b_2))$
  4. multiply together

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Belief propagation

$$t_1 \to b_1 \to x_1 \leftarrow b_2 \leftarrow t_2$$

Features of algorithm:

- **Sequential:** first x_1, then b_2, then t_2
- **Local:** uses only "nearby" variables, determined by DAG
  - x_1 given b_1
  - b_2 given b_1 and x_1
  - t_2 given b_2
- Uses both **hard** (t_1 & b_1) and **hypothetical** (x_1, b_2, t_2) info
  - and uses them interchangeably for updates

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Limited belief propagation

Propose people propagate info through **limited** paths in DAG

- At each step, updates using only **subset** of DAG & evidence
- Accommodates **limited/no contingent thinking**
  e.g. **never** conditions on hypthetical info
- Accommodates limited number of logical steps
  e.g. considers path from hard info **only if ≤ 2 long**

Formalize model of limited propagation beliefs and show

- accommodates evidence on failure of contingent thinking
- how they relate to existing models of non-Bayesian updating
- failures of contingent thinking cause correlation neglect and dynamic inconsistency
- "microfounded" by version of propagation algorithm

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With limitations above, the procedure is as follows:

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1. x_1 is ≤ 2 steps from t_1 & b_1;
   so $\hat{p}(x_1|t_1, b_1) = p(x_1|b_1)$
2. b_2 is ≤ 2 steps from b_1 but receives no flows from x_1;
   hence $\hat{p}(b_2 \mid x_1, b_1, t_1) = p(b_2)$
3. nothing ≤ 2 steps from t_2, so $\hat{p}(t_2|b_2, x_1, b_1, t_1) = p(t_2)$

Combining

$$\hat{p}(b_2, x_1, t_2|t_1, b_1) = p(x_1|b_1, t_1)p(b_2)p(t_2)$$

which implies under-appreciation of 2's strategy (Winner's curse).

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Obvious dominance (Li, 2017) suggests that ascending auction easier to play than second price

Capture this by assuming bidder also observes b_2. Then:

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DAG factorization

- A set $N = \{1, ..., n\}$ indexes random variables $X_1, \ldots, X_n$
- A **directed acyclic graph (DAG)** consists of a set of nodes (vertices) N and a set of directed edges R with no cycles
- Distribution q factorizes according to R if

$$q(x_1, \ldots, x_n) = \prod_{i \in N} q\left(x_i \mid x_{R(i)}\right) \equiv q_R(x_1, \ldots, x_n)$$

  where for every node i, $R(i) = \{j \in N \mid (j, i) \in R\}$

- A distribution q is consistent with the DAG R if $p = p_R$.
  - when q is inconsistent with R, q_R distorts q
  - E.g., when R is $1 \to 2 \to 3$,

$$q_R(x) = q(x_1)q(x_2 \mid x_1)q(x_3 \mid x_2)$$

    even if $X_3 \not\perp X_1 | X_2$

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Model: primitives

- The DM wants to predict $X_1, \ldots, X_n$
- $G \subset N$ represents the set of **given** variables that the DM **observes or chooses**

- Let p be the "true" distribution of the variables that factorizes according to the "true" DAG R
  - Assume that p has full and finite support.

Model: parameters

- The **flows** $F_{j \to i} \subset R$ of info, (possibly empty) subset of paths in R from j to i along which update of X_j propagates to X_i
  - Any path from j to k in $F_{j \to i}$ is also in $F_{j \to k}$
  - Depends on G; make explicit only when comparing different Gs
  - Captures both how far and to which others DM propagates info
  - Examples of rules for $F_{j \to i}$:
    - all paths in R from j to i of length ≤ 2 when $i \in G$, $\emptyset$ o/w
    - all paths in R from j to i with all arrows toward i, no others
    - all paths in R from j to i with cost less than K_i
- A linear **order** $\succ$ on N in which variables updated
  - nodes in G come first: $g \in G$ and $i \notin G$ implies $i \succ g$
  - for every $i \notin G$, the variables that flow to i are

$$M(i) = \{j \in N : j \prec i \ \& \ F_{j \to i} \neq \emptyset\}$$

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Auction revisited

The "true" DAG R with givens $G = \{t_1, b_1\}$:

$$t_1 \longrightarrow b_1 \longrightarrow x_1 \longleftarrow b_2 \longleftarrow t_2$$

Natural order: $t_1 \prec b_1 \prec x_1 \prec b_2 \prec t_2$

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Auction revisited

The "true" DAG R with givens $G = \{t_1, b_1, b_2\}$:

$$t_1 \longrightarrow b_1 \longrightarrow x_1 \longleftarrow b_2 \longleftarrow t_2$$

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LIMITED PROPAGATION BELIEFS

Definition

For $\{F_{j\to k}\}_{j,k}$ and $\succ$, $\hat{p}$ is a **limited propagation belief (LPB)** if

$$\hat{p}\left(x_{N-G} \mid x_G\right) = \prod_{i\in N-G} p_{\cup_j F_{j\to i}}\left(x_i \mid x_{M(i)}\right)$$

for every x_N.

In auction,

$$\hat{p}(b_2, x_1, t_2 | t_1, b_1) = p_{t_1\to b_1\to x_1}(x_1 \mid b_1, t_1) p_{b_1\to x_1\leftarrow b_2}(b_2 \mid b_1) p_{\emptyset}(t_2)$$

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$$p\left(x_{N-G} | x_G\right) = \prod_{i \in N-G} p_R\left(x_i \mid x_{\{j \in N | i \succ j\}}\right)$$

When considering X_i, limited propagation beliefs instead

1. Use **subjective flows** into i ($\cup_j F_{j \to i}$) instead of R
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When/why depart from Bayes

Model focuses on some features that make updating hard:

1. v-colliders, e.g. $1 \to 2 \leftarrow 3$
   - $X_1 \perp X_3$ but $X_1 \not\perp X_3 | X_2$
   - Why are all the single people mean?
   - Comes up in many "pivot" problems: $b_1 \to x_1 \leftarrow b_2$
2. length of reasoning chains
   - harder to apply information over longer chains of implications
3. distinction between hard vs hypothetical info
   - Contingent thinking easier when contingency known
   - 2nd price sealed bid vs ascending auction

Features of structure of the relationships between variables, not probabilities or properties of event revealed (cf Ortoleva 2012, 2024; Gennaioli & Shleifer 2010; Zhao 2022; etc)

Experimental evidence on contingent thinking

- **Monty Hall** (Friedman 1998)
- **Acquire a company** (Samuelson & Bazerman 1985; Martinez-Marquina et al [MNV] 2019)

LPB: SB and MH have similar DAGs, featuring v-collider

1. Monty Hall: [prize] $\rightarrow$ [door opened] $\leftarrow$ [door picked]
2. SB: [value of company] $\rightarrow$ [accept/reject] $\leftarrow$ p
3. MNV: [L accept/reject] $\leftarrow$ p $\rightarrow$ [H accept/reject]

explains evidence when flow from i to j nonempty only if j adjacent to i

Experimental evidence on contingent thinking

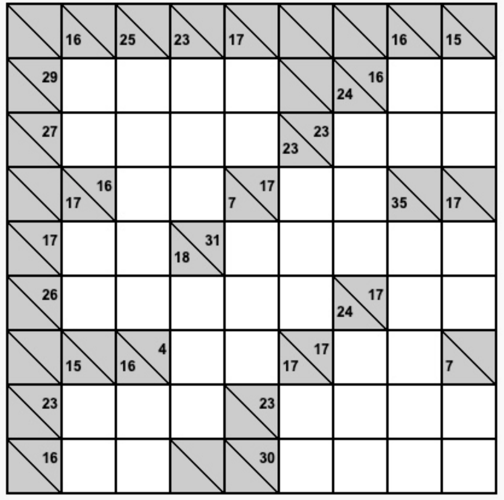
- **Auctions** (Kagel et al 1987; Li 2017)
- **Jury voting** (Esponda & Vespa, 2014/2024)
- **Sequential vs Simultaneous Markets** with informative price (Ngangoue & Weizsacker, 2021)
- **Public good provision** (Calford & Cason, 2024)

LPB: very similar DAGs to the auction example

e.g. in Jury voting, true DAG is
signal $\leftarrow$ ball $\rightarrow$ [other's vote] $\rightarrow$ winner $\leftarrow$ [subject's vote]

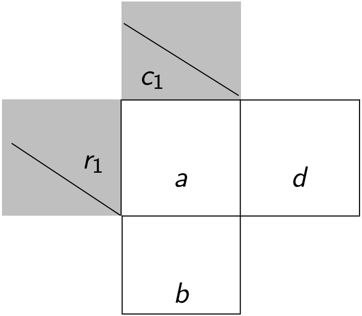
all are explained with flows are as in auction example
more flows when it's a given Details

Example: Kakuro/Cross-sum



(from kakuroconquest.com)

Example: Kakuro



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- You know r_1 and c_1. Do you prefer hint "$a = k$" or "$b = k$"?
-
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- LPB: $G = \{r_1, c_1, a\}$ vs $G' = \{r_1, c_1, b\}$

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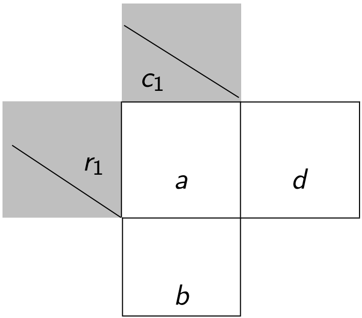

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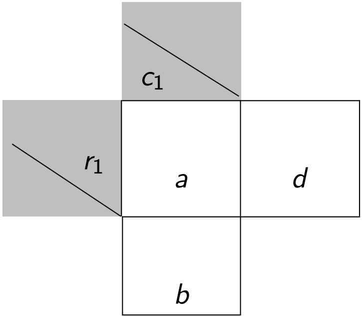

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$$R = b \to c_1 \leftarrow a \to r_1 \leftarrow d$$

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- Economic interpretation: interconnected markets

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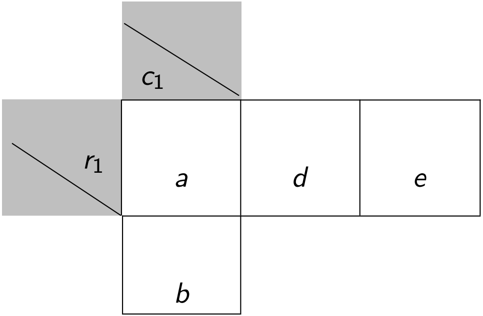
Extends to probabilistic reasoning:



So $a + b = c_1$ and $a + d + e = r_1$

You again know r_1 and c_1. Do you prefer hint "$a = k$" or "$b = k$"?

Failure of probabilistic reasoning, not just deterministic logic (cf Lipman 1999, Jakobsen 2020 or Piermont & Zuazo-Garin 2025)

Result: Bayesian updating

If flows missing an edge between non-givens, then LPB $\neq$ Bayes

Proposition

If there exists $(i,j) \in R - (F_{i\to j} \cup F_{j \to i})$ *and* $i,j \notin G$, *then* $\hat{p}(\cdot|x_G) \neq p(\cdot|x_G)$ *for almost all* p *and any ordering.*

If $\succ$ *extends* R *and* $(i,j) \in F_{i\to j}$ *for every* $(i,j) \in R$ *with* $j \notin G$, *then* $\hat{p}(\cdot|x_G) = p(\cdot|x_G)$ *for every* p.

Some contingent thinking is necessary for Bayesian updating

Enough contingent thinking is sufficient for Bayesian updating

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If $\succ$ extends R and $(i,j) \in F_{i\to j}$ for every $(i,j) \in R$ with $j \notin G$, then $\hat{p}(\cdot|x_G) = p(\cdot|x_G)$ for every p.

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Bayesian updating: necessity proof

- Suppose that the flows leave out the link in R between non-givens X_i and X_j
- Consider a p so that X_i is correlated with X_j and (X_i, X_j) independent of $X_{N-\{i,j\}}$
- For the LPB given p, X_i and X_j are independent of each other
- So $\hat{p}(x_i, x_j|x_G) = \hat{p}(x_i|x_G)\hat{p}(x_j|x_G) \neq p(x_i, x_j) = p(x_i, x_j|x_G)$
- Extend to almost all p using standard results (e.g. Transversality Theorem)

General feature: lack of contingent thinking (missing link between non-givens) leads to correlation neglect

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Failures of contingent thinking

Definition

Say that **flows exhibit no contingent thinking** if $F_{i \to j} = \emptyset$ whenever $i \notin G$.

DM does not propagate any information from hypotheticals

Implication 1: Correlation neglect

Implication 2: Failure of iterated expectations Details

Other non-rational updating models

Spiegler (2016) argues many existing non-rational expectations models have beliefs of the form

$$\tilde{p}(x_{N-G}|x_G) = p_Q(x_{N-G}|x_G)$$

for a fixed DAG Q, including

- Eyster & Rabin (2005) (when fully cursed)
- Jehiel & Koessler (2008)
- Esponda (2008)

When do limited propagation beliefs have a single DAG representation?

Example: No single DAG

Consider $G = \{t, a_1\}$ and

$$R = \begin{array}{ccccc} t & \leftarrow & \theta & & \\ & & \downarrow & \searrow & \\ a_1 & \rightarrow & a_2 & \rightarrow & a_3 \end{array}$$

Flows from t/a_1 to i be the shortest path(s) between them,
a_2 flows to a_3 and only a_3,
and no other variables flow at all.
For any ordering so that a_3 goes last, $\hat{p}(\theta, a_2, a_3|t, a_1)$ equals

$$p(\theta|t, a_1)p(a_2|t, a_1)\sum_k p(a_3|\theta = k, a_2)p(\theta = k|t)$$

Proposition

for almost every p, $\hat{p}(\theta, a_2, a_3|t, a_1) \neq p_Q(\theta, a_2, a_3|t, a_1)\ \forall Q$

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Result: Single DAG

Flows satisfy the **all-or-nothing property** if for every $i \neq j$, either $F_{i \to j} = \emptyset$ or $F_{i \to j}$ equals all paths in R from i to j

Proposition

If flows satisfy the all-or-nothing property, then then there exists a DAG Q so that $\hat{p}(x_{N-G} \mid x_G) = p_Q(x_{N-G} \mid x_G)$ for all x_N

- $Q(i) = M(i)$ for all i
- Q typically **not a subset** of R & may "reverse causality"

- All-or-nothing must hold whenever there is at most one path between any two nodes, as in auction example

Conclusion

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If flows satisfy the all-or-nothing property, then then there exists a DAG Q so that $\hat{p}(x_{N-G} \mid x_G) = p_Q(x_{N-G} \mid x_G)$ for all x_N

- $Q(i) = M(i)$ for all i
- Q typically **not a subset** of R & may "reverse causality"

- All-or-nothing must hold whenever there is at most one path between any two nodes, as in auction example

Conclusion

Result: Single DAG

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Proof

- Let $R_i = \cup_j F_{j \to i}$ and note that $R_i(j) \subseteq R(j)$ for all j
- Reorder so that: nodes in R_i come first & $R_i(j) \subset \{1, ..., j-1\} \equiv 1 : j-1$ for all j
- If $p(x_{1:n}) = p_{R_i}(x_{1:n})$ then $p_{R_i}(x_i | x_{M(i)}) = p(x_i | x_{Q(i)})$
- Assume (IH) $p(x_{1:j-1}) = p_{R_i}(x_{1:j-1})$
- To show $p(x_{1:j}) = p_{R_i}(x_{1:j})$, let $L = R(j) \setminus R_i(j)$. Then

$$\begin{aligned} p(x_{1:j}) &= p(x_j \mid x_{R(j)}) p(x_{1:j-1}) \\ &= \sum_{x_L} p(x_j \mid x_{R_i(j)}, x_L) p(x_L | x_{1:j-1}) p(x_{1:j-1}) \end{aligned}$$

- By IH, equality holds when $X_L \perp X_{1:j-1}$.
- If not, then there is a "backdoor path" from $j' < j$ to L
- But all-or-nothing forces this path into R_i, hence no backdoor path and the claim holds

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Application: public good game

- 3 player, 3 period game
- DM is Player 1; chooses effort a_1 after observing signal t
- Player $\tau > 1$ chooses whether to veto ($a_\tau = 0$) in period τ
- Public good gives benefit if 1 exerts effort and neither vetos
- state θ determines distribution of costs of good (NIMBY)
- Player 1 has utility

$$u(x) = a_1(\theta a_2 a_3 - c),$$

  $c > 0$ (known)
- Players 2 and 3 observe an almost perfect signal about θ and the action of their immediate predecessor
  - Each vetos only if predecessor took $a_\tau = 1$ and wrong state
- Player 2 prefers $\theta = 1$, Player 3 prefers $\theta = 0$
- All variables take values in $\{0, 1\}$

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- Conditional distributions:
  - $p(\theta = 1) = \frac{1}{2}$
  - $p(t = \theta \mid \theta) = q \in \left(\frac{1}{2}, 1\right)$
  - $p(a_2 = 1 | \theta, a_1) \approx \theta a_1$
  - $p(a_3 = 1 | \theta, a_2) \approx (1 - \theta) a_2$

  ($\approx$ to ensure full support)
- **Unique** sequential equilibrium for any c: no effort ($a_1 = 0$) intuition: at least one future player vetos, so why bother?

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- Solution concept with LPB/DAG: ε-personal equilibrium where $\varepsilon \to 0$ (Spiegler 2016)

- With single DAG beliefs, unique equilibrium also has no effort

  Intuition: $p(a_3 = 1 | x_{Q(a_3)}) > 0$ only if $\{\theta, a_2\} \subset Q(3)$, $\theta = 0$, and $a_2 = 1$, which implies $\theta a_2 a_3 = 0$

Application: public good game

- Assume Player 1 has LPB with givens $G = \{t, a_1\}$ and

$$R = \begin{array}{ccccc} t & \leftarrow & \theta & & \\ & & \downarrow & \searrow & \\ a_1 & \rightarrow & a_2 & \rightarrow & a_3 \end{array}$$

  same flows as before

- Player 1's beliefs are $\hat{p}(x_1, x_2, x_3 | x_t, x_a)$ which equals

$$p(\theta|t, a_1)p(a_2|t, a_1)\sum_k p(a_3|\theta = k, a_2)p(\theta = k|t)$$

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- High effort in equilibrium with LPB when $c < q^2(1-q)$
- Three terms:

$$p(\theta = 1 \mid t = 1, a_1) \approx q$$

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- Combining,

$$\hat{p}(\theta a_2 a_3 = 1 | t = 1, a_1 = 1) \approx q^2(1-q)$$

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Takeaways

- Limited propagation beliefs have economically meaningful differences from other boundedly rational models of updating
- LPB explain (some) evidence on contingent thinking and suggest new directions
- Contingent thinking necessary for Bayesian updating and dynamic consistency; lack of it impies correlation neglect
- In paper:
    - Algorithmic "foundation"
    - Testable implications
    - More flows or information $\not\Longrightarrow$ higher utility
    - Applications to social learning

Thank you

Experimental evidence on contingent thinking

Monty Hall (Friedman 1998)

1. prize is behind door v
2. subject picks door d
3. Host opens door $o \neq v, d$
4. Subjects offered chance to switch doors.

Subjects should switch

Evidence: Most don't

$v \rightarrow o \leftarrow d$

LP: explains evidence when $F_{i \rightarrow j}$ contains forward links but not backward links or only links adjacent to j

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Acquire a company (Samuelson & Bazerman 1985; Martinez-Marquina et al [MNV] 2019)

1. company values self at $v \in \{v_h, v_l\}$ (equally likely and $v_l < v_h < 2v_l$)
2. subject values company at $\frac{3}{2}v$ (unknown) and offers price p
3. company decides whether to accept & get p or not & get v

MNV consider two companies, known to be v_h and v_l

Subjects should offer $p = v_l$

Evidence: most don't, more likely to do so in MNV

$v \to d \leftarrow p$ (SB) vs $d_l \leftarrow p \to d_h$ (MNV)
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Jury voting (Esponda & Vespa, 2014/2023):

1. Ball $\theta \in \{W, B\}$ drawn and determines t_3 (probabilistically)
2. v_2 is vote of computer which matches θ.
3. v_1 is never observed and equals B with probability 1.
4. e is the winner of the election, majority rule.
5. Subject observes t_3 and chooses $v_3 \in \{W, B\}$
   (EV get rid of signal to simplify)

Subjects should vote W (pivotal only if $v_2 = W = \theta$ since $v_1 = B$)

Evidence: Most don't, but more likely to do so in treatments where v_2 is observed
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Experimental evidence

Auctions (Kagel & Levin 1986; Li 2017)

Evidence: sealed bid 2nd price auctions played differently than English (ascending) auction
overbidding in common value auction

LP: Essentially same up to relabeling as Jury voting

Back

Correlation neglect

Proposition

If flows exhibit no contingent thinking, then the limited propagation beliefs for any p set X_i independent of X_j given X_G for every distinct $i, j \in N - G$.

non-contingent thinkers neglect correlation

ITERATED EXPECTATIONS

For two sets of givens $G \subset G'$, we are interested in comparing

$$\hat{p}(x|x_G) \;\&\; \hat{p}(x|x_{G'}).$$

- In paper, analyze the case where $G' = G \cup \{g^*\}$
- For simplicity in talk, $G = \{1\}$, $R(1) = \emptyset$, and $G' = \{1, 2\}$

Assumptions:

1. The flows from i only depend on whether or not i is a given

   Formally: $F^{G}_{i \to j} = F^{G'}_{i \to j}$ for all $i \neq 2$

2. Any flow from 2 through 1 is also a flow from 1

   Formally: If $F^{G'}_{2 \to k}$ contains a path from 1 to k, then that path also belongs to $F^{G}_{1 \to k}$

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ITERATED EXPECTATIONS

Say that $\hat{p}$ satisfies **iterated expectations** if for every objective distribution p and every x_N,

$$\hat{p}\left(x_{N-G'}|x_G\right) = \sum_{x_{g^*}} \hat{p}(x_{g^*}|x_G)\hat{p}\left(x_{N-G'}|x_G, x_{g^*}\right). \tag{1}$$

Tight connection with dynamic consistency, Blackwell monotonicity, recursive utility, etc.

implied by Bayesian updating

Iterated Expectations: necessity

Proposition

Suppose that flows exhibit no contingent thinking.

If $\hat{p}$ satisfies iterated expectations, then $F_{2\to j}^{G'}$ contains a path that does not intersect 1 for at most one $j \in N - G'$.

If 2 influences two or more variables,

- $\hat{p}$ neglects correlation between them when given G
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- and DM realizes it when 2 is a given

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Iterated Expectations: sufficiency

Reverse implication depends on what 2 affects when given

- If 2 flows to nothing, then iterated expectations holds
- If 2 flows to exactly one variable, then iterated expectations may or may not hold
  characterization of exactly which is in paper
  back